Introductory note to *1961/?

This text is based on a shorthand draft from late 1961 or the subsequent years for a lecture that was never given. The manuscript was found in an envelope from the American Philosophical Society postmarked 13 December 1961, and labelled by Gödel "Vortrag, Konzept". Gödel was elected to membership in that society in April 1961 and signed the roll book the following November. After the November meeting he wrote to G.W. Corner, the executive secretary of the Society, "I would first like to say that I enjoyed the meeting on November 9 very much and that some of the lectures were very interesting to me." On December 13 Corner then wrote Gödel to advise him of the custom that new members were invited to give twenty-minute talks on topics of their own choosing, and he suggested that Gödel could do so either in April or November 1963. Since this letter of Corner's was found in the envelope with Gödel's manuscript, it is reasonable to assume that the latter is a draft for such a lecture. However, there is no indication that Gödel ever replied to the invitation.

The aim of the lecture is to describe in philosophical terms the development of the study of the foundations of mathematics in our century and fit it into a general scheme of possible philosophical *Weltanschauungen*. Gödel places the *Weltanschauungen* and also specific philosophical views on a scale according to their affinity with or distance from metaphysics or religion. On the left, furthest from metaphysics, he places skepticism, materialism, positivism, empiricism and pessimism, on the right, spiritualism, idealism, theology, apriorism and optimism. Gödel notes that the development of philosophy since the Renaissance largely has been from the right to the left on this scale.

Gödel makes clear that his sympathies lie with the right. This is in keeping with his statements to various interlocutors; compare the picture of Gödel's world view that one obtains from *Wang 1987*. This sympathy is muted and qualified in Gödel's publications and lectures; in the present text, Gödel gives more open expression to this attitude than in anything else intended for public presentation. Still, Gödel says that "the truth lies in the middle or consists of a combination of the two conceptions" (page 7), and much of the lecture is a development of that thought.

Gödel observes that mathematics is a discipline that long has withstood the *Zeitgeist*'s move to the left; in particular, Mill's attempt to develop an empirical theory of mathematics never found much support. Also, mathematics became more abstract and reached more clarity concerning its foundations, e.g., through the exact founding of the infinitesimal calculus and of the complex numbers.

Around the turn of the century mathematics got into trouble. In particular, the antinomies of set theory were seized upon by the skeptics and used as a pretext for a more "leftist" view on mathematics. However, Gödel regards their reaction as unwarranted, for two reasons: (1) the contradictions do not occur within mathematics, but on its extreme border towards philosophy; and (2) they have been resolved in a manner that is completely satisfactory and which seems almost obvious to everyone who understands the theory.

However, Gödel's incompleteness theorem shows that the rightward conception of mathematics as a complete system of truths cannot be rescued by appeal to a set of axioms and formal rules. On the other hand, he also rejects the leftward conception of mathematics according to which

> the truth of the axioms from which mathematics starts out cannot be justified or recognized in any way, and therefore the drawing of consequences from them has meaning only in a hypothetical sense, whereby this drawing of consequences itself (in order to satisfy even further the spirit of the time) is construed as a mere game with symbols according to certain rules, likewise not ⟦supported by⟧ insight ⟦page 5⟧.

Gödel inveighs against this leftist conception of mathematics because it

> gives up aspects whose fulfillment would in any case be very desirable and which have much to recommend themselves: namely, on the one hand, to safeguard for mathematics the certainty of its knowledge, and on the other, to uphold the belief that for clear questions posed by reason, reason can also find clear answers ⟦page 6⟧.

Expressing his view that "the truth lies in the middle", Gödel retains the rightward idea of mathematics as a system of truth, which is complete in the sense that "every precisely formulated yes-or-no question in mathematics must have a clear-cut answer." However, he rejects the idea, shared by rightists and leftists alike, that the basis of these truths is their derivability from axioms. He points out that Hilbert's attempt to combine these two ideas proves to be impossible in view of his own incompleteness theorem and the corollary concerning consistency proofs. (Characteristically, Gödel does not mention that these results are his own.) Instead of trying to secure the certainty of mathematics by manipulation of physical symbols, Gödel proposes "cultivating (deepening) knowledge of the abstract concepts themselves which lead to the setting up of these mechanical systems" (page 7).

This alternative approach to the foundations of mathematics is connected with the second reason Gödel gave for not being disturbed by the antinomies of set theory, *viz.*, that they have been resolved in a manner that is completely satisfactory and which seems almost obvious to everyone who understands the theory. According to Gödel, there is a certain kind of obviousness that accrues to mathematical statements once their meaning has been properly clarified. This clarification cannot merely consist in giving explicit definitions of some concepts in terms of others, but requires, at least to a large extent, an elucidation of meaning that does not appeal to other, ultimately undefinable concepts.

The method for this clarification of meaning Gödel finds in Husserl's phenomenology. Gödel describes Husserl's method as "focusing more sharply on the concepts concerned by directing our attention ... onto our own acts in the use of these concepts, onto our powers in carrying out our acts, etc." It is "a procedure or technique that should produce in us a new state of consciousness in which we describe in detail the basic concepts we use in our thought, or grasp other basic concepts hitherto unknown to us" (page 8).

Gödel sees no objective reason for rejecting phenomenology and presents as a consideration in its favor that the development of children proceeds in two directions. They do not only experiment with the objects of the external world and develop their sensory and motor organs. They also come to a better understanding of language and of the basic concepts on which it rests, such as that of a logical inference. Gödel observes that if one views the development of empirical science as a systematic extension of what the child does when it develops in the first direction, then given the astonishing success of this procedure, it seems quite possible that a systematic and conscious advance in the second direction will also far exceed the expectations one may have a priori (pages 8–9).

The central piece of support that Gödel gives in favor of his view is the following example of a development in the second direction:

> ... in the systematic establishment of the axioms of mathematics, new axioms, which do not follow by formal logic from those previously established, again and again become evident. It is not at all excluded by the negative results mentioned earlier that nevertheless every clearly posed mathematical yes-or-no question is solvable in this way. For it is just this becoming evident of more and more new axioms on the basis of the meaning of the primitive notions that a machine cannot imitate [[page 9]].

Except for the reference to the incompleteness results, which were attained after Husserl had developed his phenomenology, this is all very

similar to what Husserl wrote about mathematical evidence. Gödel states that it also agrees in principle with Kant's conception of mathematics (page 9).[a]

This is all Gödel says in this manuscript concerning what the phenomenological method consists in and the reasons why we should accept it. It may be the paucity of particulars and argument in this part of the manuscript that made him decide not to give the lecture. However, Gödel had a very thorough knowledge of phenomenology and there is much more to be found on phenomenology elsewhere in his writings.

Gödel did not start to study Husserl until 1959 (*Wang 1981*, page 658 and *Wang 1987*, page 12 ff.), but he soon became quite absorbed by the reading of Husserl's writings. He owned all of Husserl's main works,[b] and his underlining and comments in the margins indicate that

[a]In remarking that "ever newer axioms that are logically independent from the earlier ones" must be "intuitively grasped", Gödel says that this is in general agreement with Kant's view of mathematics. He says that Kant's claims taken literally are false

> since Kant asserts that in the derivation of geometrical theorems we always need new geometrical intuitions, and that therefore a purely logical derivation from a finite number of axioms is impossible [[p. 9]].

Gödel no doubt has in mind the well-known passage in the *Critique of pure reason* where Kant discusses the proof that the sum of the angles of a triangle is two right angles and remarks that the theorem is arrived at "through a chain of inferences guided throughout by intuition" (A 717/B 745). The interpretation of this as implying that the theorem cannot be derived logically from the axioms is not obvious but is central to some influential modern interpretations of Kant's conception of the role of intuition in mathematics, in particular in *Beth 1956/7*; *Hintikka 1965*, *1967*, and other writings; and *Friedman 1985*, *1992*. These interpretations emphasize, as Gödel does not in his remark, that the formal logic available to Kant is much weaker than modern logic. Gödel's partial agreement with this line of interpretation is noteworthy; at the time of working on the present text, he could have known Beth's paper but in all probability not the writings of Hintikka.

Note that the word translated "intuitions" in the above quotation from Gödel is "Intuitionen" and not the Kantian "Anschauungen", no doubt because the intuitions in question are propositional, and Kant uses "Anschauung" in relation to *objects*. The question of the relation of Kant's and Gödel's conceptions of intuition is a complicated one. Gödel certainly recognized that they were not identical.

[b]Gödel owned the *Logische Untersuchungen* (in the edition *1968*), *Ideen*, Book I (*1950–* , volume III, first edition), *Cartesianische Meditationen und Pariser Vorträge* (*1950–* , volume I, second edition), *Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie* (*1950–* , volume VI, second edition). He also owned *Husserl 1965* and both volumes of *Spiegelberg 1965*. All of these books are heavily annotated by Gödel, with the exception of *Husserl 1968*. In the latter were found, however, several pages of Gödel's shorthand notes, referring to page numbers in the text, indicating that Gödel probably had based his study of that text on a borrowed copy. The work was out of print for many years before 1968. The notes indicate that the text Gödel had used was the second edition (*1913a*, *1921*). References to Husserl in the following will be, as usual, to page and line in *Husserl 1950–*

he studied them carefully. Most of his comments are positive and expand upon Husserl's points. However, sometimes he is critical, notably in his notes to the *Logische Untersuchungen* (*Husserl 1900, 1901*) and to some sections of Husserl's last work, the *Krisis* (*Husserl 1950–* , Volume VI, one part published in 1936). Generally, Gödel is most appreciative of the *Ideen* (*1913*) and the other works written after Husserl's "idealist" conversion around 1907.

Gödel had expressed views on the philosophy of mathematics similar to those of Husserl long before he started to study him. What he found in Husserl was not radically different from his own view; what impressed him seems to have been Husserl's general philosophy, which would provide a systematic framework for a number of his own earlier ideas on the foundations of mathematics.

Two main points in Husserl's philosophy of mathematics that were central to Gödel even before he read Husserl are the following:

Realism. Gödel had held realist views on mathematical entities since his student days (*Wang 1974*, pages 8–11), or more exactly, since 1921–22.[c] It has often been noted that his essay "Russell's mathematical logic" (*1944*) contains a number of strongly realist statements, such as the well-known passage in which he says that the assumption of classes and concepts is in the same sense necessary to obtain a satisfactory system of mathematics as physical bodies are necessary for a satisfactory theory of our sense perceptions (page 137).

Gödel expresses similar views in his Gibbs lecture (**1951*) and also in his essay "Is mathematics syntax of language?" (**1953/9*, both in this volume). In the latter he writes the following about *concepts* and their properties:

> Mathematical propositions, it is true, do not express physical properties of the structures concerned [[in physics]], but rather properties of the *concepts* in which we describe those structures. But this only shows that the properties of those concepts are something quite as objective and independent of our choice as physical properties of matter. This is not surprising, since concepts are composed of primitive ones, which, as well as their properties, we can create as little as the primitive constituents of matter and their properties [[**1953/9-V*, page 9]].

[c] *Feferman 1984a*, pp. 549–552; see, however, also Feferman's introductory note to *Gödel *1933o* in this volume, p. 39, where Feferman comments on Gödel's apparently skeptical statement about Platonism on p. 19 of that manuscript.

Note the comparison between concepts and physical objects in this passage and in that from *1944* referred to above. Gödel does not say straightforwardly that the properties of concepts are objective, but that they are *as objective as* the physical properties of matter. This brings him close to Husserl, who in the *Ideen* and his other "idealist" works maintained that physical objects, as well as concepts and mathematical objects, are objective, but not in the straightforward, realist sense. Husserl called himself an "idealist". However, his view is much more realist and Platonist than traditional idealist points of view, and as Husserl later acknowledged, "idealism" is a very misleading label for his position.[d]

In a letter of 3 February 1959, responding to an inquiry from Paul Arthur Schilpp about the state of his contribution **1953/9* to his volume on Carnap (*Schilpp 1963*), Gödel says:

> ... a complete elucidation of the situation turned out to be more difficult than I had anticipated, doubtless in consequence of the fact that the subject matter is closely related to, and in part identical with, one of the basic problems of philosophy, namely the question of the *objective reality of concepts* and their relations [[emphasis mine]].

Also in his Supplement to the second edition of "What is Cantor's continuum problem?" (*1964*, page 272) Gödel compares mathematical and physical objects and notes that the question of the objective existence of the objects of mathematical intuition is an exact replica of the question of the objective existence of the outer world. He then goes into the issue of mathematical intuition. This issue is intimately connected with the question of the nature and existence of mathematical entities. On this point, too, Gödel's views are similar to those of Husserl, and the similarity may help us to see what they both are getting at.

Intuition. Already in *1944*, Gödel talks about elementary mathematical evidence or mathematical "data" and compares it to sense perception (page 128; cf. pages 137, 142). Gödel's own notion of mathematical intuition is discussed in all the other three papers mentioned, and in each of them it is compared to perception. First, in his Gibbs lecture, Gödel states:

[d] In 1934 he wrote in a letter to Abbé Baudin: "No ordinary 'realist' has ever been as realistic and concrete as I, the phenomenological 'idealist' (a word which by the way I no longer use)." Quoted in *Kern 1964*, p. 276 n., my translation.

> What is wrong, however, is that the *meaning* of the terms (that is, the concepts they denote) is asserted to be something man-made and consisting merely in semantical conventions. The truth, I believe, is that these concepts form an objective reality of their own, which we cannot create or change, but only *perceive* and describe [[**1951*, page 30, emphasis mine]].

In his essay for the Carnap volume (**1953/9-V*, pages 6–7), Gödel writes:

> The similarity between mathematical *intuition* and a *physical sense* is very striking. It is arbitrary to consider "This is red" an immediate datum, but not so to consider the proposition expressing modus ponens or complete induction (or perhaps some simpler propositions from which the latter follows). For the difference, as far as it is relevant here, consists solely in the fact that in the first case a relationship between a concept and a particular object is perceived, while in the second case it is a relationship between concepts [[emphasis mine]].

Finally, there is the famous passage from *1964* (page 271) where Gödel compares mathematical intuition to sense perception and says that we have something like a *perception* of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true.

Gödel leaves it unclear here whether he thinks that the objects of mathematical intuition are propositions, as in the quotation from **1953/9*, concepts (first quotation, from **1951*), sets and concepts (as would naturally be inferred from the passage in *1964*, page 271), or all three. An answer emerges from Husserl. According to him, intuition, as well as perception, is of objects. There are two kinds of intuition, according to Husserl: perception, where the object intuited is a physical object, and categorial, or eidetic intuition, where the object is an abstract entity. The object, whether it be concrete or abstract, is always intuited as having various properties and bearing relations to other objects. These properties and relations can be singled out for our attention in acts of judgment, in which the object is judged to have such and such features. Intuition of objects is hence more basic for Husserl than judgments: intuition provides the evidence for judgments. It is not clear whether Gödel shared this view that intuition of objects is more basic.

Neither perception nor categorial intuition is an infallible source of evidence. Both always involve anticipations concerning aspects of their objects that have not yet been explored, and which may turn out to be wrong. Since there is no other source of evidence, Husserl acknowledges

that errors are always possible,[e] even in mathematics and logic.[f]

Skeptics may be doubtful that we can experience any other objects than physical objects. However, Husserl points out that already in the perception of a physical object many features are involved that go beyond mere sensation, thus, e.g., the reification that is involved in the notion of object itself. These features, and especially those involved in reification, are those that are studied in mathematics. Gödel seems to have something very similar in mind in when he writes in the supplement to *1964*:

> That something besides the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively different from sensations or mere combinations of sensations, e.g., the idea of object itself .... Evidently the "given" underlying mathematics is closely related to the abstract elements contained in our empirical ideas. It by no means follows, however, that the data of this second kind, because they cannot be associated with actions of certain things upon our sense organs, are something purely subjective, as Kant asserted. Rather they, too, may represent an aspect of objective reality, but, as opposed to the sensations, their presence in us may be due to another kind of relationship between ourselves and reality [[pages 271–2]].

This goes far beyond what Gödel says about Husserl in the present manuscript. However, Gödel's very high praise of Husserl in this manuscript indicates that we may expect a high degree of similarity in their views.[g]

Gödel's characterization of the relationship between Kant and Husserl is interesting. Here, as in the case of the rightist and the leftist views on mathematics, where Gödel found the truth in the middle, he recognizes "deep truths" in the old view, and seeks to extricate these insights from their unclear and incorrect formulations. This extrication and further development of Kant's fundamental insights Gödel finds in Husserl:

[e]See *Husserl 1913*, §87 (*1950–* , III/1, 201.34–35).

[f]E.g., in *Husserl 1929*, §58 (*1950–* , XVII, 164.32–34), also *1950–* , VI, 270.28–29 (§73).

[g]This is confirmed through Gödel's very extensive remarks in his copies of Husserl's books.

> In particular, the whole phenomenological method, as I sketched it above, goes back in its [[central]] idea to Kant, and what Husserl did was merely that he first formulated it more precisely, made it fully conscious and actually carried it out for particular domains. Indeed, just from the terminology used by Husserl, one sees how positively he himself values his relation to Kant.
>
> . . . just because of the lack of clarity and the literal incorrectness of many of Kant's formulations, quite divergent directions have developed out of Kant's thought—none of which, however, really did justice to the core of Kant's thought. This requirement seems to me to be met for the first time by phenomenology, which, entirely as intended by Kant, avoids both the death-defying leaps of idealism into a new metaphysics as well as the positivistic rejection of all metaphysics. But now, if the misunderstood Kant has already led to so much that is interesting in philosophy, and also indirectly in science, how much more can we expect it from Kant understood correctly? [[pages 9–10]]

Gödel's comments in the margins of Husserl's works indicate that he regarded many of Husserl's observations as important insights. A central point is the one he speaks of in the passage above: Husserl's very special version of idealism, which incorporates both the constructivist idea that we "constitute" reality and the Platonistic idea that this reality, the mathematical as well as the physical, is there independently of us and is not "something purely subjective, as Kant asserted" (*Gödel 1964*, page 272, quoted above).

In spite of this disagreement, Gödel closes the present text with remarks about Kant that are a striking expression of the idea that the truth lies in the middle between "rightward" and "leftward" conceptions. One might have expected him to choose rather Leibniz as a point of reference in the past, as he in effect did in *1944*. Yet Gödel says that the phenomenological method goes back in its central idea to Kant and that phenomenology is the first philosophy really to do justice to "the core of Kant's thought". It may be that the study of Husserl made Gödel more sympathetic toward Kantian philosophy; however, the question of Gödel's understanding of and attitude toward Kant is a large question which has not been much explored.[h]

Dagfinn Føllesdal[i]

Transcription of **1961/?* is by Cheryl Dawson in consultation with Eckehart Köhler and William Craig. The translation was drafted by Eckehart Köhler and Hao Wang and revised by John Dawson, Charles Parsons and William Craig.

[h]On some particular aspects of this matter, however, see §§6–8 of the introductory note to **1946/9* in this volume.

[i]I am grateful to Charles Parsons for numerous suggestions for improvements and for contributing footnote a. I am also grateful to Solomon Feferman, Cheryl Dawson, John Dawson, Richard Tieszen, Dag Prawitz, Per Martin-Löf, Dick Haglund, and Herman Ruge Jervell for their valuable comments on earlier versions of this introductory note and to the Axel o Margaret Ax:son Johnson Foundation for support of this and other work.

The modern development of the foundations of mathematics in the light of philosophy (*1961/?)

Ich möchte hier versuchen, die Entwicklung der mathematischen Grundlagenforschung seit etwa der Jahrhundertwende in philosophischen Begriffen zu beschreiben und in ein allgemeines Schema von möglichen philosophischen Weltanschauungen einzuordnen. Dazu ist es zunächst nötig sich über dieses Schema selbst klar zu werden. Ich glaube, das fruchtbarste Prinzip, um eine Übersicht über die möglichen Weltanschauungen zu gewinnen, werden wird, daß man sie nach dem Grad und der Art ihrer Affinität zu, bzw. Abkehr von, der Metaphysik (oder Religion) einteilt. So bekommt man sofort eine Einteilung in zwei Gruppen: auf deren einer Seite Materialismus, Skeptizismus, Positivismus, auf deren anderer, Spiritualismus, Idealismus, Theologie stehen, und man sieht auch sofort Gradunterschiede in dieser Reihe, indem der Skeptizimus noch weiter von der Theologie steht als der Materialismus, währenddes anderseits der Idealismus, z. B. in seiner pantheistischen Form, eine Abschwächung der Theologie im eigentlichen Sinn ist.

Aber auch für die Analyse speziell zulässiger philosophischer Lehren, erweist sich dieses Schema als fruchtbar, indem man sie entweder in diesem Sinn einordnet oder in Mischfällen ihre materialistischen und spiritualistischen Elemente aufsucht. So wird man z. B. sagen, daß der| Apriorismus im 2
Prinzip auf die rechte und der Empirismus auf die linke Seite gehören. Aber anderseits gibt es auch solche Mischformen wie eine empiristisch begründete Theologie. Man sieht ferner auch, daß der Optimismus im Prinzip nach rechts und der Pessimismus nach links gehört. Denn der Skeptizismus ist ja ein Pessimismus hinsichtlich der Erkenntnis. Der Materialismus neigt dazu, die Welt als einen ungeordneten und daher sinnlosen Haufen von Atomen zu betrachten. Außerdem erscheint ihm der Tod als endgültige und vollständige Vernichtung, während anderseits Theologie und Idealismus in Allem Sinn, Zweck und Vernunft sehen—anderseits ist der Schopenhauersche Pessimismus eine Mischform, nämlich ein pessimistischer Idealismus. Ein anderes Beispiel einer offentsichtlich rechtsseitigen Theorie ist die des objektiven Rechts und objektiver aesthetischer Werte, während die Deutung der Ethik und Aesthetik auf Grund von Gewohnheit, Erziehung, etc. nach links gehört.

Nun ist es eine bekannte Tatsache, es ist auch eine Platitüde, daß die Entwicklung der Philosophie seit der Renaissance im großen und ganzen von rechts nach links gegangen ist, nicht in einer geraden Linie, sondern mit Rückschlag; aber doch im ganzen. Insbesondere hat diese Entwick-

The modern development of the foundations of mathematics in the light of philosophy (*1961/?)

I would like to attempt here to describe, in terms of philosophical concepts, the development of foundational research in mathematics since around the turn of the century, and to fit it into a general schema of possible philosophical world-views [*Weltanschauungen*]. For this, it is necessary first of all to become clear about the schema itself. I believe that the most fruitful principle for gaining an overall view of the possible world-views will be to divide them up according to the degree and the manner of their affinity to or, respectively, turning away from metaphysics (or religion). In this way we immediately obtain a division into two groups: skepticism, materialism and positivism stand on one side, spiritualism, idealism and theology on the other. We also at once see degrees of difference in this sequence, in that skepticism stands even farther away from theology than does materialism, while on the other hand idealism, e.g., in its pantheistic form, is a weakened form of theology in the proper sense.

The schema also proves fruitful, however, for the analysis of philosophical doctrines admissible in special contexts, in that one either arranges them in this manner or, in mixed cases, seeks out their materialistic and spiritualistic elements. Thus one would, for example, say that apriorism belongs in principle on the right and empiricism on the left side. On the other hand, however, there are also such mixed forms as an empiristically grounded theology. Furthermore one sees also that optimism belongs in principle toward the right and pessimism toward the left. For skepticism is certainly a pessimism with regard to knowledge. [Moreover,] materialism is inclined to regard the world as an unordered and therefore meaningless heap of atoms. In addition, death appears to it to be final and complete annihilation, while, on the other hand, theology and idealism see sense, purpose and reason in everything. On the other hand, Schopenhauer's pessimism is a mixed form, namely a pessimistic idealism. Another example of a theory evidently on the right is that of an objective right and objective aesthetic values, whereas the interpretation of ethics and aesthetics on the basis of custom, upbringing, etc., belongs toward the left.

Now it is a familiar fact, even a platitude, that the development of philosophy since the Renaissance has by and large gone from right to left—not in a straight line, but with reverses, yet still, on the whole. Particularly

lung in der Physik gerade in unserer Zeit einen Höhepunkt erreicht, indem weitgehend die Möglichkeit einer Erkenntnis der objektivierbaren Sachverhalte bestritten [wird,] und [es] behauptet wird, daß man sich | begnügen muß, Beobachtungsresultate vorauszusagen; was eigentlich das Ende jeder theoretischen Wissenschaft im üblichen Sinne ist (obwohl dieses Voraussagen für praktische Zwecke wie Fernsehapparaten oder Atombomben vollkommen hinreichend sein kann).

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Es müßte ein Wunder sein, wenn diese, ich möchte sagen rabiate, Entwicklung sich nicht auch in der Auffassung der Mathematik geltend gemacht hätte. Tatsächlich hat die Mathematik ihrer Natur nach als aprioriische Wissenschaft selbst überall eine Neigung nach rechts und hat daher dem Zeitgeist, der seit der Renaissance herrschte, lang widerstanden, d. h., die empiristische Theorie der Mathematik, wie sie z. B. von Mill aufgestellt wurde, fand keinen rechten Anklang. Ja, die Mathematik selbst entwickelt sich zu immer höheren Abstraktionen weg von der Materie und zu immer größerer Klarheit in den Grundlagen (etwa durch exakte Begründung der Infinitesimalrechnung, der komplexen Zahlen) also weg vom Skeptizismus.

Aber schließlich um die Jahrhundertwende hatte auch ihre Stunde geschlagen, insbesondere waren es die Antinomien der Mengenlehre, Widersprüche, die angeblich innerhalb der Mathematik aufgetreten waren, welche in ihrer Bedeutung von Skeptikern und Empiristen übertrieben und zum Vorwand für den Umsturz | nach links verwendet wurden. Ich sagte "*angeblich*" und "übertrieben", weil (1) diese Widersprüche nicht in der Mathematik sondern an ihrer äußersten Grenze zur Philosophie zu auftraten und (2) sie in einer vollständig befriedigenden sowie für jeden, der diese Theorie versteht, beinahe selbstverständlichen Weise aufgelöst wurden. Aber solche Argumente helfen nichts gegen den Zeitgeist und so war das Resultat, daß von vielen oder den meisten Mathematikern geleugnet wurde, daß die Mathematik, wie sie sich vorher entwickelt hatte, ein System von Wahrheiten darstellt; vielmehr wurde das nur für einen (je nach Temperament größeren oder kleineren) Teil der Mathematik anerkannt und alles übrige wurde bestenfalls in einem hypothetischen Sinn beibehalten, indem nämlich die Theorie mit Recht nur besagt, daß man aus gewissen (nicht zu rechtfertig[end]en) Annahmen mit Recht gewisse Folgerungen ziehen kann. Dabei schmeichelte man sich, daß man eigentlich alles wesentliche beibehalten hätte. [Indem ja was den Mathematiker interessiert neben dem Folgern aus jenen Annahmen, ist, was man vollziehen kann.] In Wahrheit aber wird dadurch die Mathematik zu einer empirischen Wissenschaft, denn wo ich etwa aus den willkürlich angenommenen Axiomen beweise, daß jede natürliche Zahl Summe von vier Quadraten ist, so folgt daraus gar nicht mit Sicherheit, daß ich nie ein Gegenbeispiel gegen dieses Theorem finden werde, denn meine Axiome könnten | ja widerspruchsvoll sein und ich kann höchstens sagen, es folge mit einer gewissen Wahrscheinlichkeit, da ja trotz vieler Folgerungen bisher kein Widerspruch entdeckt wurde. Außerdem

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in physics, this development has reached a peak in our own time, in that, to a large extent, the possibility of knowledge of the objectivizable states of affairs is denied, and it is asserted that we must be content to predict results of observations. This is really the end of all theoretical science in the usual sense (although this predicting can be completely sufficient for practical purposes such as [making] television sets or atom bombs).

It would truly be a miracle if this (I would like to say rabid) development had not also begun to make itself felt in the conception of mathematics. Actually, mathematics, by its nature as an a priori science, always has, in and of itself, an inclination toward the right, and, for this reason, has long withstood the spirit of the time [*Zeitgeist*] that has ruled since the Renaissance; i.e., the empiricist theory of mathematics, such as the one set forth by Mill, did not find much support. Indeed, mathematics has evolved into ever higher abstractions, away from matter and to ever greater clarity in its foundations (e.g., by [giving] an exact foundation of the infinitesimal calculus [and] the complex numbers)—thus, away from skepticism.

Finally, however, around the turn of the century, its hour struck: in particular, it was the antinomies of set theory, contradictions that allegedly appeared within mathematics, whose significance was exaggerated by skeptics and empiricists and which were employed as a pretext for the leftward upheaval. I say "*allegedly*" and "exaggerated" because, in the first place, these contradictions did not appear within mathematics but near its outermost boundary toward philosophy, and secondly, they have been resolved in a manner that is completely satisfactory and, for everyone who understands the theory, nearly obvious. Such arguments are, however, of no use against the spirit of the time, and so the result was that many or most mathematicians denied that mathematics, as it had developed previously, represents a system of truths; rather, they acknowledged this only for a part of mathematics (larger or smaller, according to [their] temperament) and retained the rest at best in a hypothetical sense—namely, [one] in which the theory properly asserts only that from certain assumptions (not themselves to be justified), we can justifiably draw certain conclusions. They thereby flattered themselves that everything essential had really been retained. [Since, after all, what interests the mathematician, in addition to drawing consequences from these assumptions, is what can be carried out.] In truth, however, mathematics becomes in this way an empirical science. For if I somehow prove from the arbitrarily postulated axioms that every natural number is the sum of four squares, it does not at all follow with certainty that I will never find a counterexample to this theorem, for my axioms could after all be inconsistent, and I can at most say that it follows with a certain probability, because in spite of many deductions no contra-

verlieren durch diese hypothetische Auffassung der Mathematik viele Fragen die Form: Gilt der Satz A oder gilt er nicht? Vollkommen im Sinn dann von willkürlichen Annahmen kann ich ja nicht erwarten, daß sie die sonderbare Eigenschaft haben, immer gerade entweder A oder $\sim A$ zu implizieren.

Obwohl diese nihilistischen Folgerungen dem Zeitgeist sehr gut entsprechen, so setzte hier eine Reaktion, selbstverständlich nicht seitens der Philosophie, wohl aber seitens der Mathematik ein, die, wie ich ja schon sagte, ihrer Natur nach dem Zeitgeist gegenüber sehr widerspenstig ist. Und so entstand jenes merkwürdige Zwitterding, welches sowohl dem Zeitgeist als der Natur der Mathematik gerecht zu werden suchte, welches der Hilbertsche Formalismus darstellt. Dieser besteht in folgendem: es wird einerseits in Übereinstimmung mit den in der heutigen Philosophie herrschenden Ideen anerkannt, daß die Axiome, von denen die Mathematik ausgeht, sich in ihrer Wahrheit in keiner Weise begründen oder erkennen lassen und daher das Ziehen von Folgerung[en] aus ihnen nur in einem hypothetischen Sinn Bedeutung hat, wobei man dieses Folgern selbst (um dem Zeitgeist noch weiter Genüge zu tun) als ein bloßes Spiel mit Symbolen nach gewissen, ebenfalls nicht einsichtigen, Regeln auffaßt.

Anderseits aber hielt man an dem der früheren rechtsseitigen Philosophie der Mathematik und dem Instinkt des Mathematikers entsprechenden Glauben fest, daß[a] ein Beweis für die Richtigkeit eines solchen Satzes, wie der Darstellbarkeit von allen Zahlen als Summen [von] Quadraten, eine sichere Begründung für diesen Satz geben muß; und ferner auch daran, daß jede präzis formulierte Ja- oder Nein-Frage in der Mathematik eine eindeutige Antwort haben muß, d. h., also man geht darauf aus, für an sich unbegründete Regeln des Zeichenspiels als eine ihnen sozusagen zufällig zukommende Eigenschaft zu beweisen, daß von zwei Sätzen A, $\sim A$ immer genau einer abgeleitet werden kann. Daß nicht beide abgeleitet werden können, beinhaltet die Widerspruchsfreiheit, und daß immer einer wirklich abgeleitet werden kann, bedeutet eindeutige Beantwortbarkeit der durch A ausgedrückten mathematischen Frage. Natürlich muß man, wenn man diese
6 beiden Aussagen mit mathematischer Sicherheit begründen will, | einen gewissen Teil der Mathematik im Sinne der alten rechtsseitigen Philosophie als wahr anerkennen. Aber das ist ein Teil, der dem Zeitgeist viel weniger zuwider ist als die hohen Abstraktionen der Mengenlehre. Er bezieht sich nämlich nur auf konkrete und endliche Objekte im Raum, nämlich die Zeichenkombinationen.

[a]The word *man* has been omitted here between *daß* and *ein*. See the last section of textual notes for this article.

diction has so far been discovered. In addition, through this hypothetical conception of mathematics, many questions lose the form "Does the proposition A hold or not?" [For,] from assumptions construed as completely arbitrary, I can of course not expect that they have the peculiar property of implying, in every case, exactly either A or $\sim A$.

Although these nihilistic consequences are very well in accord with the spirit of the time, here a reaction set in—obviously not on the part of philosophy, but rather on that of mathematics, which, by its nature, as I have already said, is very recalcitrant in the face of the *Zeitgeist.* And thus came into being that curious hermaphroditic thing that Hilbert's formalism represents, which sought to do justice both to the spirit of the time and to the nature of mathematics. It consists in the following: on the one hand, in conformity with the ideas prevailing in today's philosophy, it is acknowledged that the truth of the axioms from which mathematics starts out cannot be justified or recognized in any way, and therefore the drawing of consequences from them has meaning only in a hypothetical sense, whereby this drawing of consequences itself (in order to satisfy even further the spirit of the time) is construed as a mere game with symbols according to certain rules, likewise not [supported by] insight.

But, on the other hand, one clung to the belief, corresponding to the earlier "rightward" philosophy of mathematics and to the mathematician's instinct, that a proof for the correctness of such a proposition as the representability of every number as a sum of four squares must provide a secure grounding for that proposition; and furthermore, also that every precisely formulated yes-or-no question in mathematics must have a clear-cut answer. I.e., one thus aims to prove, for inherently unfounded rules of the game with symbols, as a property that attaches to them so to speak by accident, that of two sentences A [and] $\sim A$, exactly one can always be derived. That not both can be derived constitutes consistency, and that one can always actually be derived means that the mathematical question expressed by A can be unambiguously answered. Of course, if one wishes to justify these two assertions with mathematical certainty, a certain part of mathematics must be acknowledged as true in the sense of the old rightward philosophy. But that is a part that is much less opposed to the spirit of the time than the high abstractions of set theory. For it refers only to concrete and finite objects in space, namely the combinations of symbols.

Was ich bisher sagte sind ja eigentlich nur Selbstverständlichkeiten, an die ich bloß erinnern wollte, weil sie für das Folgende wichtig sind. Aber der nächste Schritt in der Entwicklung ist nun der, daß sich zeigt, [daß] es nicht möglich ist, die alten rechtsseitigen Aspekte der Mathematik auf eine solche mit dem Zeitgeist mehr oder weniger in Übereinstimmung stehende Weise zu retten. Sogar wenn man sich auf die Theorie der natürlichen Zahlen beschränkt, ist es unmöglich, ein System von Axiomen und formalen Regeln zu finden, aus dem für jeden zahlentheoretischen Satz immer entweder A oder $\sim A$ ableitbar wäre, und es ist ferner unmöglich, für einigermaßen umfassende Axiome der Mathematik einen Widerspruchsfreiheitsbeweis durch bloße Betrachtung der konkreten Zeichenkombinationen ohne Einführung abstrakterer Elemente zu führen. Die Hilbertsche Kombination von Materialismus und den Aspekten der klassischen Mathematik erweist sich also als unmöglich.

Daher bleiben nur zwei Möglichkeiten offen; man muß entweder die alten rechtsseitigen Aspekte der Mathematik aufgeben oder man muß versuchen, diese im Widerspruch mit dem Zeitgeist aufrechtzuerhalten. Offenbar ist der erste Weg der einzige, der in unsere Zeit paßt und der daher auch meistenteils eingeschlagen wird. Aber man sollte sich bewußt bleiben, daß das eine rein negative Haltung ist. Man gibt einfach Aspekte auf, deren Erfüllung doch jedenfalls sehr wünschenswert wäre und die viel für sich haben, nämlich einerseits der Mathematik die Sicherheit ihrer Erkenntnisse zu wahren und anderseits den Glauben aufrechtzuerhalten, daß für von der Vernunft gestellte klare Fragen die Vernunft auch klare Antworten finden kann. Und man beachte, man gibt diese Aspekte auf, nicht weil die erzielten mathematischen Resultate dazu zwingen, sondern weil das die einzige Möglichkeit ist, trotz dieser Resultate mit der herrschenden Philosophie in Übereinstimmung zu bleiben.

Nun kann man sich ja den großen Fortschritten, welche unsere Zeit in vieler Hinsicht aufzuweisen hat, keineswegs verschließen und man kann mit einem gewissen Recht geltend machen, daß diese Fortschritte ebendiesem
7 Linksgeist in der Philosophie und Weltanschauung zu danken seien. | Aber anderseits, wenn man die Sache in richtiger historischer Perspektive betrachtet, muß man sagen, daß die Fruchtbarkeit des Materialismus zum Teil nur auf den Exzessen und der falschen Richtung der vorhergehenden rechtsseitigen Philosophie beruht. Was Recht und Unrecht bzw. Wahrheit und Falschheit dieser beiden Richtungen betrifft, so scheint mir die richtige Einstellung die zu sein, daß Wahrheit in der Mitte liegt oder in einer Kombination dieser beiden Auffassungen besteht.

Nun hatte ja im Fall der Mathematik gerade Hilbert eine solche Kombination versucht, aber diese offenbar zu primitiv und zu stark nach einer Richtung hinneigend. Jedenfalls ist kein Grund, dem Zeitgeist blindlings zu vertrauen und es ist daher zweifellos der Mühe wert, einmal die andere der oben genannten Alternativen, welche die genannten Resultate übriglassen,

What I have said so far are really only obvious things, which I wanted to recall merely because they are important for what follows. But the next step in the development is now this: it turns out that it is impossible to rescue the old rightward aspects of mathematics in such a manner as to be more or less in accord with the spirit of the time. Even if we restrict ourselves to the theory of natural numbers, it is impossible to find a system of axioms and formal rules from which, for every number-theoretic proposition A, either A or $\sim A$ would always be derivable. And furthermore, for reasonably comprehensive axioms of mathematics, it is impossible to carry out a proof of consistency merely by reflecting on the concrete combinations of symbols, without introducing more abstract elements. The Hilbertian combination of materialism and aspects of classical mathematics thus proves to be impossible.

Hence, only two possibilities remain open. One must either give up the old rightward aspects of mathematics or attempt to uphold them in contradiction to the spirit of the time. Obviously the first course is the only one that suits our time and [is] therefore also the one usually adopted. One should, however, keep in mind that this is a purely negative attitude. One simply gives up aspects whose fulfillment would in any case be very desirable and which have much to recommend themselves: namely, on the one hand, to safeguard for mathematics the certainty of its knowledge, and on the other, to uphold the belief that for clear questions posed by reason, reason can also find clear answers. And as should be noted, one gives up these aspects not because the mathematical results achieved compel one to do so but because that is the only possible way, despite these results, to remain in agreement with the prevailing philosophy.

Now one can of course by no means close one's eyes to the great advances which our time exhibits in many respects, and one can with a certain justice assert that these advances are due just to this leftward spirit in philosophy and world-view. But, on the other hand, if one considers the matter in proper historical perspective, one must say that the fruitfulness of materialism is based in part only on the excesses and the wrong direction of the preceding rightward philosophy. As far as the rightness and wrongness, or, respectively, truth and falsity, of these two directions is concerned, the correct attitude appears to me to be that the truth lies in the middle or consists of a combination of the two conceptions.

Now, in the case of mathematics, Hilbert had of course attempted just such a combination, but one obviously too primitive and tending too strongly in one direction. In any case there is no reason to trust blindly in the spirit of the time, and it is therefore undoubtedly worth the effort [at least] once to try the other of the alternatives mentioned above—which the results cited leave open—in the hope of obtaining in this way a

zu versuchen in der Erwartung, auf diese Weise eine brauchbare Kombination zu erhalten. Das heißt offenbar die Sicherheit der Mathematik nicht dadurch sicherzustellen, daß man gewisse Eigenschaften in Projektion auf materielle Systeme, nämlich das Umgehen mit physischen Symbolen, beweist, sondern dadurch, daß man die Erkenntnis der abstrakten Begriffe selbst, welche zur Aufstellung jener mechanischen Systeme führt, kultiviert (vertieft) und daß man ferner nach dem gleichen Verfahren Einsichten über die Lösbarkeit und über tatsächliche Methoden zur Lösung aller sinnvollen mathematischen Probleme zu gewinnen sucht.

Auf welche Weise aber ist es möglich, die Kenntnis jener abstrakten Begriffe zu erweitern, d. h., also diese Begriffe selbst zu präzisieren und umfassende und sichere Einsicht über die für sie bestehenden Grundrelationen, d. h., die für sie geltenden Axiome, zu gewinnen? Offenbar nicht dadurch oder jedenfalls nicht ausschließlich dadurch, daß man versucht, explizite Definitionen für Begriffe und Beweise für Axiome zu geben. Denn dann braucht man ja dafür offenbar andere undefinierbare abstrakte Begriffe und für sie geltende Axiome. Sonst hätte man ja nichts, woraus man definieren oder beweisen könnte. Das Verfahren muß also wenigstens zum
8 großen Teil | in einer Sinnklärung bestehen, die nicht in Definieren besteht.

Nun gibt es ja heute den Beginn einer Wissenschaft, welche behauptet, eine systematische Methode für eine solche Sinnklärung zu haben, und das ist die von Husserl begründete Phänomenologie. Die Sinnklärung besteht hier darin, daß man die betreffenden Begriffe schärfer ins Auge faßt, indem man die Aufmerksamkeit in einer bestimmten Weise dirigiert, nämlich auf unsere eigenen Akte bei der Verwendung dieser Begriffe, auf unsere Mächte bei der Vollführung unserer Akte, etc. Man muß sich dabei klar darüber sein, daß diese Phänomenologie nicht eine Wissenschaft im selben Sinn ist wie die anderen Wissenschaften. Sie ist vielmehr [oder sollte jedenfalls sein] ein Verfahren oder Technik, welches in uns einen neuen Bewußtseinszustand hervorbringen soll, in dem wir die von uns verwendeten Grundbegriffe unseres Denkens detaillieren oder andere bisher uns unbekannte Grundbegriffe erfassen. Ich glaube, es besteht gar kein Grund, ein solches Verfahren von vornherein als aussichtslos abzulehnen. Am wenigsten Grund dafür haben natürlich Empiristen, denn das würde heißen, daß ihr Empirismus in Wahrheit ein Apriorismus mit dem verkehrten Vorzeichen ist.

Aber nicht nur besteht kein objektiver Grund der Ablehnung ⟦der Phänomenologie⟧, sondern im Gegenteil, man kann Gründe zugunsten angeben. Wenn man die Entwicklung eines Kindes betrachtet, so sieht man, daß diese in zwei Richtungen vor sich geht; einerseits besteht sie in einem Experimentieren mit den Gegenständen der Außenwelt und mit seinen Sinnen- und Bewegungsorganen, anderseits in einem besseren und besseren Verstehenlernen der Sprache und das heißt, sobald das Kind über die primitivste Form des Bezeichnens hinaus ist, der Grundbegriffe auf denen sie beruht. Hinsichtlich der Entwicklung in dieser zweiten Richtung kann man

workable combination. Obviously, this means that the certainty of mathematics is to be secured not by proving certain properties by a projection onto material systems—namely, the manipulation of physical symbols—but rather by cultivating (deepening) knowledge of the abstract concepts themselves which lead to the setting up of these mechanical systems, and further by seeking, according to the same procedures, to gain insights into the solvability, and the actual methods for the solution, of all meaningful mathematical problems.

In what manner, however, is it possible to extend our knowledge of these abstract concepts, i.e., to make these concepts themselves precise and to gain comprehensive and secure insight into the fundamental relations that subsist among them, i.e., ⟦into⟧ the axioms that hold for them? Obviously not, or in any case not exclusively, by trying to give explicit definitions for concepts and proofs for axioms, since for that one obviously needs other undefinable abstract concepts and axioms holding for them. Otherwise one would have nothing from which one could define or prove. The procedure must thus consist, at least to a large extent, in a clarification of meaning that does not consist in giving definitions.

Now in fact, there exists today the beginning of a science which claims to possess a systematic method for such a clarification of meaning, and that is the phenomenology founded by Husserl. Here clarification of meaning consists in focusing more sharply on the concepts concerned by directing our attention in a certain way, namely, onto our own acts in the use of these concepts, onto our powers in carrying out our acts, etc. But one must keep clearly in mind that this phenomenology is not a science in the same sense as the other sciences. Rather it is [or in any case should be] a procedure or technique that should produce in us a new state of consciousness in which we describe in detail the basic concepts we use in our thought, or grasp other basic concepts hitherto unknown to us. I believe there is no reason at all to reject such a procedure at the outset as hopeless. Empiricists, of course, have the least reason of all to do so, for that would mean that their empiricism is, in truth, an apriorism with its sign reversed.

But not only is there no objective reason for the rejection ⟦of phenomenology⟧, but on the contrary one can present reasons in its favor. If one considers the development of a child, one notices that it proceeds in two directions: it consists on the one hand in experimenting with the objects of the external world and with its ⟦own⟧ sensory and motor organs, on the other hand in coming to a better and better understanding of language, and that means—as soon as the child is beyond the most primitive designating ⟦of objects⟧—of the basic concepts on which it rests. With respect to the development in this second direction, one can justifiably say that the child

mit Recht sagen, daß das Kind Bewußtseinszustände verschiedener Höhen durchläuft; z. B. kann man sagen, daß ein höherer Bewußtseinszustand erreicht wird, wo das Kind zuerst den Gebrauch des Wortes lernt und ebenso in dem Augenblick, wo es zum ersten Mal eine logische Schlußfolgerung versteht. Nun kann man ja die ganze Entwicklung der empirischen Wissenschaft als eine systematische und bewußte Erweiterung dessen, was das
9 Kind tut, wenn es sich in der ersten Richtung entwickelt, auffassen. | Der Erfolg dieses Verfahrens ist aber ein erstaunlicher und weitaus größerer als man a priori erwarten würde. Er führt ja zur ganzen technologischen Entwicklung der neueren Zeit. Das läßt es also als durchaus möglich erscheinen, daß auch ein systematisches und bewußtes Weitergehen in der zweiten Richtung die Erwartungen, die man a priori haben kann, weit übertreffen wird.

Tatsächlich hat man Beispiele wo, sogar ohne ein systematisches und bewußtes Verfahren anzuwenden, sondern ganz von selbst, eine beträchtliche weitere Entwicklung in der zweiten Richtung, über den "gesunden Menschenverstand" hinaus, stattfindet. Es zeigt sich nämlich, daß bei einem systematischen Aufstellen der Axiome der Mathematik immer wieder neue und neue[re] Axiome evident werden, die nicht formallogisch aus den bisher aufgestellten folgen. Es ist durch [die] früher erwähnten negativen Resultate gar nicht ausgeschlossen, daß trotzdem auf diese Weise jede klar gestellte mathematische Ja- oder Nein-Frage lösbar ist, denn eben dieses Evidentwerden immer neuerer Axiome auf Grund des Sinnes der Grundbegriffe ist etwas, was eine Maschine nicht nachahmen kann.

Ich möchte darauf aufmerksam machen, daß dieses intuitive Erfassen immer neuerer und von den früheren logisch unabhängiger Axiomen, welches zur Lösbarkeit aller Probleme selbst eines sehr eingeschränkten Gebiets nötig ist, prinzipiell mit der Kantschen Auffassung der Mathematik übereinstimmt. Allerdings sind die diesbezüglichen Äußerungen von Kant wörtlich verstanden unrichtig, denn Kant behauptet, daß wir zur Ableitung der geometrischen Theoreme immer neue geometrische Intuitionen benötigen und also eine rein logische Ableitung aus einer endlichen Zahl von Axiomen unmöglich ist. Das ist nachweislich falsch. Wenn wir aber in dieser Aussage den Term "geometrisch" durch "mathematisch" oder durch "mengentheoretisch" ersetzen, dann wird es eine nachweislich richtige Aussage. Ich glaube, es ist eine allgemeine Eigenschaft vieler Kantschen Behauptungen, daß sie wörtlich verstanden falsch sind, aber in einem allgemeineren Sinn tiefe Wahrheiten enthalten. Insbesondere geht die ganze phänomenologi-
10 sche Methode,| wie ich sie vorhin skizzierte, der Idee nach auf Kant zurück und was Husserl getan hat, war bloß, daß er sie zuerst präziser formuliert und völlig bewußt gemacht und für einzelne Gebiete wirklich durchgeführt hat. Man sieht ja schon aus der von Husserl angewendeten Terminologie, wie positiv er selbst seine Beziehung zu Kant wertet.

Ich glaube, daß gerade darauf, daß die Kantsche Philosophie letzten Endes, allerdings in einer nicht völlig klaren Weise, auf der Idee der Phäno-

passes through states of consciousness of various heights, e.g., one can say that a higher state of consciousness is attained when the child first learns the use of words, and similarly at the moment when for the first time it understands a logical inference.

Now one may view the whole development of empirical science as a systematic and conscious extension of what the child does when it develops in the first direction. The success of this procedure is indeed astonishing and far greater than one would expect a priori: after all, it leads to the entire technological development of recent times. That makes it thus seem quite possible that a systematic and conscious advance in the second direction will also far exceed the expectations one may have a priori.

In fact, one has examples where, even without the application of a systematic and conscious procedure, but entirely by itself, a considerable further development takes place in the second direction, one that transcends "common sense". Namely, it turns out that in the systematic establishment of the axioms of mathematics, new axioms, which do not follow by formal logic from those previously established, again and again become evident. It is not at all excluded by the negative results mentioned earlier that nevertheless every clearly posed mathematical yes-or-no question is solvable in this way. For it is just this becoming evident of more and more new axioms on the basis of the meaning of the primitive notions that a machine cannot imitate.

I would like to point out that this intuitive grasping of ever newer axioms that are logically independent from the earlier ones, which is necessary for the solvability of all problems even within a very limited domain, agrees in principle with the Kantian conception of mathematics. The relevant utterances by Kant are, it is true, incorrect if taken literally, since Kant asserts that in the derivation of geometrical theorems we always need new geometrical intuitions, and that therefore a purely logical derivation from a finite number of axioms is impossible. That is demonstrably false. However, if in this proposition we replace the term "geometrical" by "mathematical" or "set-theoretical", then it becomes a demonstrably true proposition. I believe it to be a general feature of many of Kant's assertions that literally understood they are false but in a broader sense contain deep truths. In particular, the whole phenomenological method, as I sketched it above, goes back in its [[central]] idea to Kant, and what Husserl did was merely that he first formulated it more precisely, made it fully conscious and actually carried it out for particular domains. Indeed, just from the terminology used by Husserl, one sees how positively he himself values his relation to Kant.

I believe that precisely because in the last analysis the Kantian philosophy rests on the idea of phenomenology, albeit in a not entirely clear

menologie beruht und eben dadurch etwas völlig neues und eben für jede echte Philosophie charakteristisches in das Denken eingeführt hat, daß gerade darauf der ungeheure Einfluß beruht, den Kant auf die ganze folgende Entwicklung der Philosophie ausübte. Es gibt ja kaum irgendeine spätere Richtung, die nicht in irgendeiner Weise sich auf Kantsche Ideen bezieht. Anderseits haben aber eben wegen der Unklarheit und im wörtlichen Sinn Unrichtigkeit vieler Kantscher Formulierungen sich ganz entgegengesetzte philosophische Richtungen aus [[dem]] Kantschen Denken entwickelt, von denen aber keine dem Kantschen Denken in seinem Kern wirklich gerecht wurde. Dieser Forderung scheint mir erst die Phänomenologie zu genügen, welche ganz im Sinne Kants sowohl dieselben Salto mortale des Idealismus in eine neue Metaphysik als auch die positivistische Ablehnung jeder Metaphysik vermeidet. Wenn nun aber schon der falsch verstandene Kant zu so vielem Interresanten in Philosophie und indirekt auch in der Wissenschaft geführt hat, wieviel mehr kann man es von dem richtig verstandenen Kant erwarten?

way, and has just thereby introduced into our thought something completely new, and indeed characteristic of every genuine philosophy—it is precisely on that, I believe, that the enormous influence which Kant has exercised over the entire subsequent development of philosophy rests. Indeed, there is hardly any later direction that is not somehow related to Kant's ideas. On the other hand, however, just because of the lack of clarity and the literal incorrectness of many of Kant's formulations, quite divergent directions have developed out of Kant's thought—none of which, however, really did justice to the core of Kant's thought. This requirement seems to me to be met for the first time by phenomenology, which, entirely as intended by Kant, avoids both the death-defying leaps of idealism into a new metaphysics as well as the positivistic rejection of all metaphysics. But now, if the misunderstood Kant has already led to so much that is interesting in philosophy, and also indirectly in science, how much more can we expect it from Kant understood correctly?